The Scope

In this series of four inertia-related articles, I will discuss quantifiable principles that govern the "playability" of a piano key action (PKA). Since a big part of playability relates directly to "inertia", I will focus heavily on defining and determining the true inertia *at the key*. This parameter is referred to herein as Inertia at the Key (IK). In this first article, I'll explain the concepts of IK,"total force", and "inertial force", and how the IK can **actually be measured** with proper equipment. This will necessarily include a brief discussion of "reflected inertia", which is so important to inertial calculations and measurements. I'll also introduce a new, measurable parameter called Key Return, which is a direct measurement of key "sluggishness". Slightly idealized versions of the major action components will then be introduced, paving the way towards achieving test-based determination of the "mass configuration" of a PKA. For a given PKA, the collection of its "idealized" components will be referred to as the idealized PKA. This article concludes with two pie charts, emanating directly from the physics, giving the proportion of IK due to the major action components.

The second "inertia" article introduces a simple and useful "see-saw" type of mathematical model. It is designed to correspond exactly to an idealized PKA in all relevant respects, but is easier to visualize. The model is then used to analyze the response of important "primary" parameters ¸ Balance Force (BF), IK, and Key Return (KR) ¸ to various changes in the PKA's mass configuration. Such manipulations of typical models clearly reveal the huge effect that hammer mass has on the IK. Similarly, they show how keylead mass in the key's front end (i.e., "front mass") has a small effect on the IK, but a potentially large effect on KR. The fourth article – and part of the third ¸ will focus more on the kinetic, noninvasive measurement capabilities of the Key Force One (KFO) machines, developed and built by the author. It will be shown how idealized PKA's can be characterized largely from these measurements, done on real PKA's, by simultaneously solving motion equations for two types of movement. With the mass configuration – and a mathematical model ¸ quickly determined for *each* note across a piano, any "deviant" notes can be intelligently and easily reconfigured to get their primary parameters in line with neighboring notes, or indeed any desired values.

For these articles, "playability" of a PKA is expressed by some combination of:

– constant-speed forces occurring at the finger
– frictional forces
– inertial forces occurring at the finger during accelerated downstrokes, and
– the readiness and ability of the keystick to rise unhindered, back toward its "rest" position, due solely to gravitational, frictional, and spring/magnetic forces, and inertial properties of the mechanism.

The spring/magnetic forces are only those that might be occuring in the stroke's "pre-let-off" (PLO) region. Springs and friction associated with escapement and resetting of the jack are not considered part of the playability discussion herein. Let-off forces and parameters will be discussed fully in a separate article. Furthermore, "key repetition" is not really the focus of these inertia articles, although several of the parameters discussed herein have a significant effect on repetition behavior.

The "Static" Forces

The term "static" as used herein does not necessarily mean stationary. Rather, it simply implies "zero acceleration". Thus, static forces are those forces that are - or should be - measured while the key is moving at constant speed. The traditional parameters of Down Weight, Up Weight, Balance Weight and Frictional Weight, obtained using "gram weight" techniques, are examples of these forces. As I described in the previous article, and in U.S. Patent 8,049,090, these parameters have many inadequacies. In response to these deficiencies, the KFO machines were designed to continuously measure the *actual* reaction forces, during a constant-speed downstroke, and again during a constant-speed upstroke. Force averages are taken for each, across the stroke's PLO region, resulting in an Average Down Force (ADF) and an Average Up Force (AUF). The average of these two is the Average Balance Force (ABF), while half the difference is the Average Frictional Force (AFF). Knowing any two of these four "static" force parameters specifies exactly the other two. These average values are very accurate and repeatable, and are also required for accurately measuring the IK. They are simply more accurate and repeatable "replacements" for the age-old "gram weight" parameters. In these articles, the term "grams-force" will be used as a *unit of force*, in keeping with terminology within the industry. The amount of force that gravity exerts at sea-level on a body of mass "x" [grams] is considered herein as "x" grams-force.

More on the "Primary" Parameters

Throughout these articles, the term Application Point (AP) signifies a point on the top surface of the keystick, approximately 12 mm from its front edge. As mentioned, the KR is a direct measurement of the time it takes the key's AP to rise unhindered from some specified, depressed position, back to a known point close to "rest" position. This depressed position should be safely within the stroke's PLO region, so that jack and balancier springs are not involved. KR is an additional, verifiable parameter, directly related to playability. It is a hybrid parameter, related to both "static" forces and inertial properties. It is one of the primary parameters, the other three being Balance Force (or Down Force), Frictional Force, and IK. IK, fully defined and described below, is the actual, "effective" mass moment of inertia of the entire mechanism *at the keystick itself*.

The Course of Development

Having degrees in Mechanical Engineering, I have attacked these problems from a strict scientific perspective. I started playing piano in a fairly serious manner nearly 15 years ago, and at some point began to examine the piano action key mechanism itself. The fact that there seemed to be no method or device out there that could determine the true, effective inertia of a piano action was the main impetus for my long, arduous path. In the course of developing a machine that could tackle this "inertia" problem, I also developed other revolutionary methods and parameters. These include:

– static force and "let-off force" measurements,
– measurement of the "jack trip" point, along the keystroke,
– "kinetic" key dip/key leveling measurements, and
– "key repetition" parameters and methods.

Moment of Inertia "at the key", as Related to Measured Forces

Imagine any multi-lever, rotating mechanism. One lever is manipulated, with the other levers responding and moving/rotating accordingly, based on the geometry of the mechanism. Generally, the mechanism will be "pre-loaded" - by gravity, springs, or magnets - so the manipulated lever is indeed free to move, with sufficient urging about its own pivot. One may rotate the lever at a constant speed, while measuring the reaction forces, to obtain an average, constant-speed "down force" (or "down torque") across the stroke. This represents the "resistance" during the stroke due solely to the pre-loading force, along with friction. If one then accelerates the lever at some known angular acceleration, while measuring the reaction torques, one then has a graph of "total torque" for that lever, corresponding to that angular acceleration. Due to inherent deflections, there will be oscillations in the "total torque" data, and thus some appropriate "average total torque" must be calculated across some portion of the movement. If the "average constant-speed down torque" is subtracted from the "average total torque", the result is the "average inertial torque" (AIT). This represents the average reaction torque on the lever, at the given angular acceleration, stemming solely from the various far-flung masses being resistant to motion. It's the reaction torque that would be experienced at that acceleration, if there were no friction, gravity, springs or magnets. By employing a rotational form of Newton's Second Law on the lever being manipulated, one can solve for the moment of inertia "at the lever" (I_{lev}) as:

$$I_{lev} = \frac{AIT}{\alpha}$$

where "Ờ" is the lever's angular acceleration. I_{lev} is a constant for the given mechanism. If the angular acceleration doubles, so does the AIT, thus keeping I_{lev} the same. I_{lev} is an *intrinsic* parameter of the mechanism, but always associated with the particular lever being excited, about its own axis. Absolutely no mass or geometry information about the mechanism must be known beforehand. Rather, the whole purpose of I_{lev} is to quantify the entire mass distribution ¸ as it relates to inertial resistance at the lever ¸ with one number!

Now assume the multi-body mechanism is a typical grand PKA, with the lever to be excited being the keystick. Assume that the excitation of the keystick is performed with a downwardly-moving "finger", acting at an AP some horizontal distance "L" from the balance rail pivot point "P". The reaction force is also measured at the AP. The "finger" excites the keystick with a downward acceleration "a", while measuring the reaction forces continuously, and averaging them across the stroke. This average reaction force is the "average total force" (ATF), which includes all the gravitational, frictional, and spring/magnetic (if applicable) forces, in addition to the inertial force created by all the accelerating mass. If the Average Down Force (ADF) is also measured, and subtracted from the ATF, the Average Inertial Force (AIF) is obtained. To a close approximation, the downward acceleration "a" is the product of "L" and "Ờ". The torque about P equals force at the AP multiplied by "L". With I_{lev} now called Inertia at the Key (IK), and AIT equaling (AIF)(L), the above equation becomes:

$$IK = \frac{(AIF)(L)^2}{a}$$

Replacing "AIF" in this equation with "ATF – ADF", and changing units slightly, the equation is stated in terms of parameters whose direct measurement is possible with the KFO. It becomes:

$$IK = \frac{(ATF - ADF)(0.00981)(L)^2}{a} \qquad \text{Eq. 1}$$

Regarding units, ATF and ADF are in grams-force, "L" in mm, "a" in mm/ms^2, and IK in gŝmm^2.

Reflected Inertia

The concept of "reflected" inertia will now be explained. Assume there are two rotating bodies, A and B, coupled together so that B's angular speed is some multiple of A's angular speed. They could be meshing spur gears, for instance. There may also be an intervening rotating body/gear between the two. Regardless, body B rotates at a speed of (G)($Ð_A$), where "G" is the "gear ratio", and $Ð_A$ is the angular speed of body A. Conversely, the torque/moment at body A is equal to "G" times the torque at body B. If body A is indeed a spur gear (Gear A), to be accelerated about its axis by a motor, one needs to know the "inertial load" of the *entire* mechanism **at Gear A**, for proper sizing of the motor for acceleration. One can easily determine the local moment of inertia of all elements rotating *with* Gear A (including the gear itself, and the rotating portion of the motor). The true, effective moment of inertia at Gear A, however, must include an *additional* term, which is due to the "downstream" component - the Local Group B. This additional term is the "reflected" moment of inertia of Local Group B - at Gear A. The Local Group B is simply the combination of Gear B and anything rotating with it. One first calculates the moment of inertia of the Local Group B - about its own "local" axis, then multiplies this by the square of the gear ratio "G".

Reflected Inertia in the Piano Action

For a grand PKA, the reflected inertia "at the key" of the various components is determined in this same manner. The sum of these reflected values is the IK. The five components contributing to the IK are the hammer shank, hammerhead, wippen, keystick with no keyleads, and the "front mass" (keyleads). The keystick may be further divided into the bare key, along with the capstan and backcheck. The keystick and "front mass" are rotating with the key, and – like Gear A of the above example – need no reflection. The hammer shank and hammerhead rotate together, at an angular speed *very* different from the keystick. The keystick and the hammer assembly can be considered as Gear A and Local Group B, respectively, in the previous section. The gear ratio, between the keystick and the hammer assembly, will be referred to as the Moment Ratio (MR) of the piano action. MR thus indicates the angular speed of the hammer assembly, relative to that of the keystick, during a downstroke. For both the shank and the hammerhead, their "local" moments of inertia are first calculated, about their local, hammer flange axis. Each value is then multiplied by the square of the MR, to obtain its "reflected" inertial contribution at the key. The "local" moment of inertia of the wippen is then calculated, and multiplied by the square of its own gear ratio, relative to the keystick. This gear ratio, expressing the angular speed of the wippen relative to the keystick, will be referred to as the Wippen Moment Ratio, "MR_{wip}". It has a typical value in a grand action of between 1.5 and 2.0.

The Idealized PKA

An idealized PKA is a PKA that has certain important "distributed" masses replaced by theoretical point masses at well-defined locations. It responds to applied finger forces – at any acceleration, exactly as its "real" counterpart does. Details on the "idealized" key, wippen and hammer assembly are now given.

<u>*the Idealized Key and Keyleads*</u>

For computation purposes, the bare keystick may be "split" into three separate pieces in the idealized PKA. The capstan and backcheck are treated as point masses, with masses m_{cap} and m_{back}, respectively, located at r_{cap} and r_{back} behind the balance rail pivot. The idealized key is shown in Figure 1. The three rectangular parts (A, B, and C) of the keystick account for common height and thickness variations in typical keysticks. Knowing the density of the wood, one then obtains the masses m_A, m_B, and m_C of the three pieces. The moment of inertia of each piece, about the pivot P, is then obtained using standard "beam type" inertia formulas, coupled with the Parallel Axis Theorem. The sum of all three is the moment of inertia "$I_{key,P}$", about P, of the bare keystick. The mass of the bare wooden keystick is "m_{key}", its center of mass at some distance "r_{key}" in front of P. The ordinate dimensions shown in Figure 1, having no arrowheads, are all measured from P. The Application Point (AP) is a distance "L" to the right of P. Two masses are also added to the idealized key, each representing one or more embedded keyleads. They have masses of "mld_1" and "mld_2", located at distances "rld_1" and "rld_2", respectively. Each of these masses may represent two or three actual keyleads, if the latter are grouped closely together. The moment of inertia about P of the combination of these two masses will be referred to as "I_{lead}". A third mass "m_{TMA}" is also shown, at location "r_{TMA}". This corresponds to a mass added *temporarily*, at the very front of the key, strictly for KFO measurement purposes, and will be explained in the next article.

the Idealized Wippen

A reasonable approximation for a typical grand piano wippen is shown in Figure 2. It consists of a uniform horizontal beam having mass m_{beam}, along with a point mass (representing the jack) of mass m_{jack} sitting near the beam's non-pivoting end. The jack mass sits at a distance r_{jack} from the pivot P, while the beam length is also approximately r_{jack}. The beam represents the two major "beams" of a typical grand action wippen. The total mass is mw_{act}, equaling the sum of m_{beam} and m_{jack}. The moment of inertia $I_{wip,loc}$ of this idealized wippen, about its own axis, is thus: $(0.333)(m_{beam})(r_{jack})^2 + (m_{jack})(r_{jack})^2$. Multiplying $I_{wip,loc}$ by the square of MR_{wip}, where MR_{wip} is the "wippen moment ratio", yields the wippen's reflected inertia at the key. AR_{wip} is the "wippen action ratio", typically in the 0.5 to 0.6 range for grand actions. If it is, say, 0.55, then one gram added to the center of mass of the wippen assembly results in an increase in BF of 0.55 grams-force. Thus, the upward force created at the AP by the idealized wippen is $(mw_{act})(AR_{wip})$ grams-force, where the mass is in grams. .

the Idealized Hammer Shank

The shank has some total mass, which includes the knuckle. The knuckle mass can be neglected, since its contribution to the shank's rotational inertia and gravitational force is minimal. The shank tends to be much wider in that same region, near the hammerflange pivot. Again, this extra mass, being very close to the axis, may be neglected. A slightly-idealized shank of mass "msh_{act}", uniform along its length, can effectively replace the actual shank. Its mass is a half-gram to one gram less than the total shank/knuckle mass. The shank's total length, as measured from its axis, is assumed for convenience to equal R_h, the distance from the hammer flange axis to the hammerhead's center of mass. The idealized shank is shown in Figure 3. The shank's moment of inertia about the hammerflange axis is $(0.333)(msh_{act})(R_h)^2$. Its reflected inertia *at the key* is therefore $(0.333)(msh_{act})(R_h)^2(MR)^2$. The "upward" static force created at the AP by the idealized shank is $(msh_{act})(AR/2)$, with mass in grams and force in "grams-force". Thus, adding 1 gram to the mass of the idealized shank increases the static force at the AP by (0.5)(AR) grams-force.

the Idealized Hammerhead

Figure 3 also shows the hammerhead, idealized as a point mass, near the very end of the shank. It is offset from the shank, to approximately coincide with the actual hammerhead's center of mass The mass of the hammerhead is referred to as mh_{act}; its center of mass is at a true distance R_h from the hammerflange axis P, as shown. The "local" moment of inertia of the hammerhead, about P, is simply $(mh_{act})(R_h)^2$. Its reflected moment of inertia at the key is then $(mh_{act})(R_h)^2(MR)^2$. The "upward" static force created at the AP by the idealized hammerhead is simply $(mh_{act})(AR)$.

While the idealized PKA allows all necessary computations to be performed, it is convenient to create a simple "leveraged see-saw" mathematical model, equivalent to the idealized PKA both statically and dynamically. This model is detailed in the next article, where the concept of Key Return will also be further described. A series of analyses will then be presented, where three different mass configurations (MC1, MC2 and MC3) of a PKA are both modelled *and* measured. The measurements (static forces, total force, IK and Key Return) were performed kinetically with a KFO machine. This provided a two-way verification of both the model and the test equipment and methods. Emanating directly from the physics, Figure 6 , detailed fully in the next article , is given here as a preview. It shows the relative contributions of the various action components towards IK, for two different PKA's. MC3 has significantly more hammerhead mass and "front mass" than MC1. In both cases, one can clearly see the overwhelming dominance that the hammer has on Inertia at the Key.

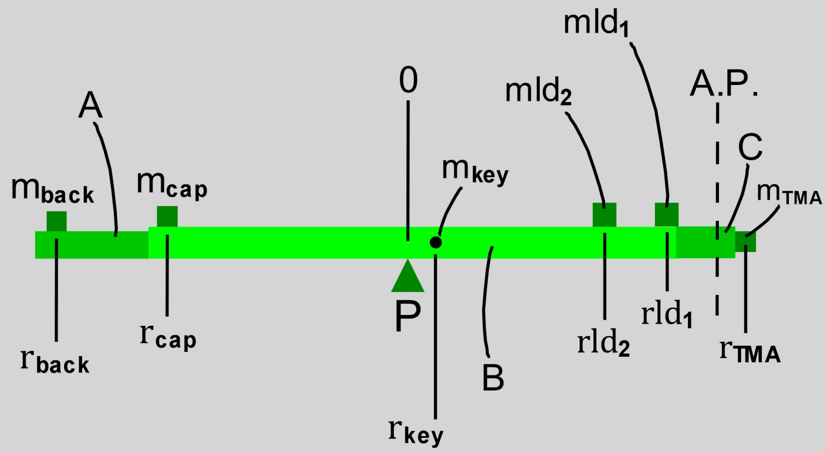

Figure 1: the Idealized Key

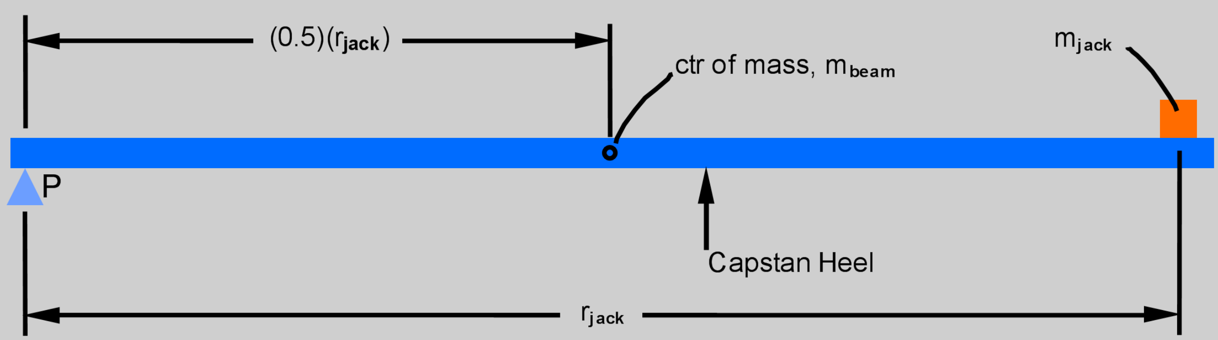

Figure 2: Idealized Wippen Assembly

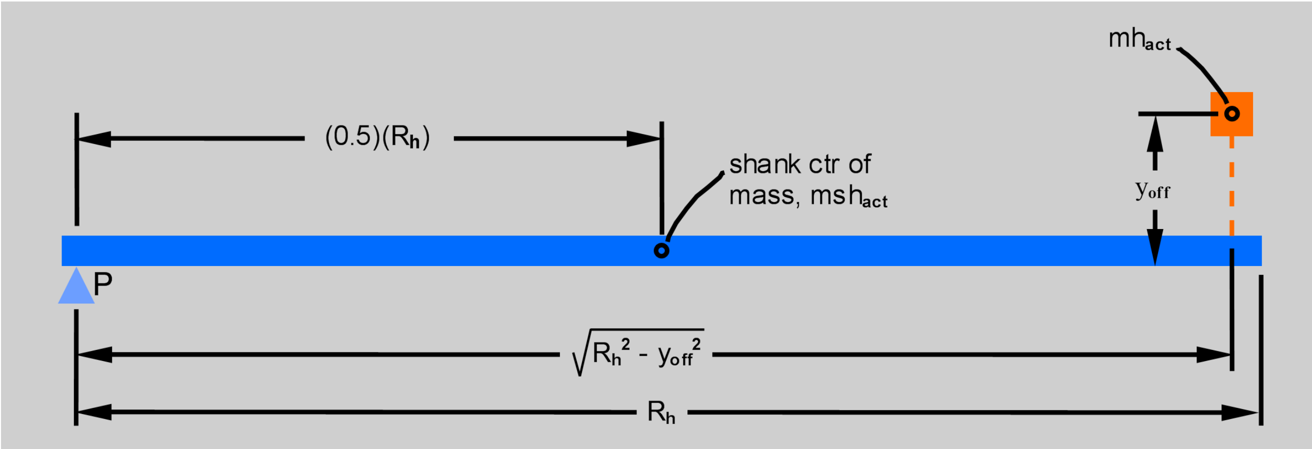

Figure 3: Idealized Hammer Assembly

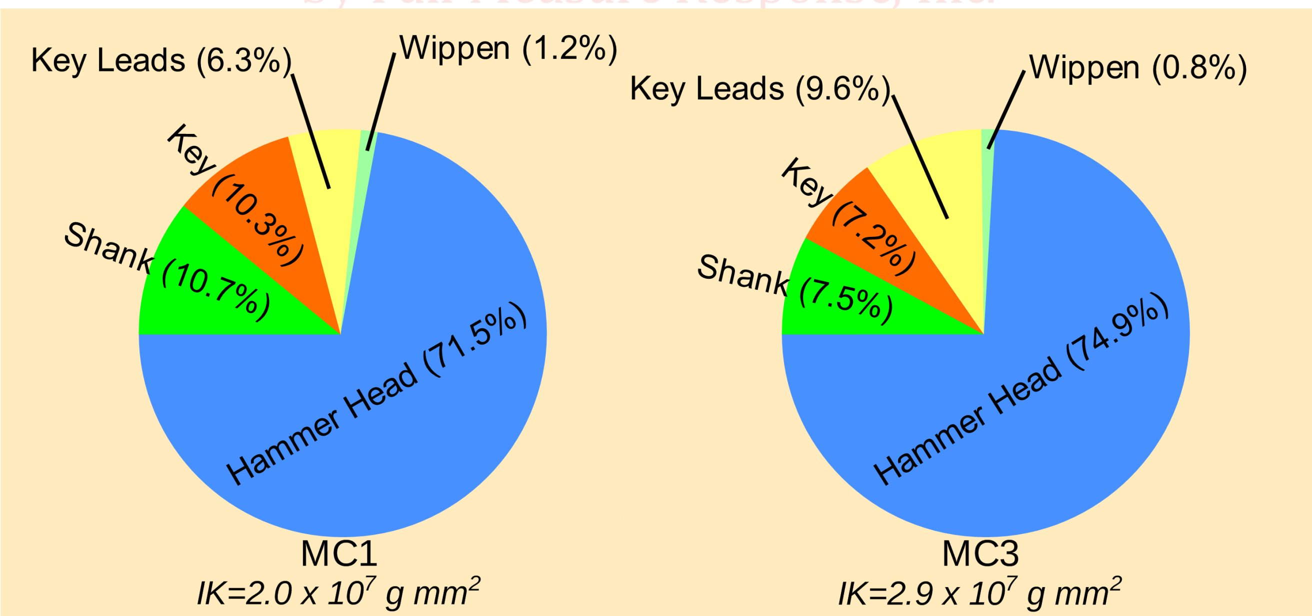

Figure 6: Inertia Contributions